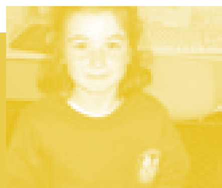
Developing computation



A project funded under the Australian Government's Numeracy Research and Development Initiative
and conducted by the Department of Education Tasmania, the Catholic Education Office (Tasmania)
and the Association of Independent Schools of Tasmania

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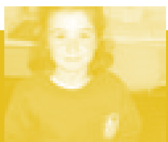
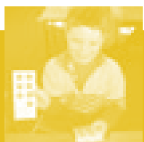
Table of Contents

Executive summary

A major policy objective of the Australian Government is to ensure that all students attain sound foundations in literacy and numeracy. In 1998, all Education Ministers agreed to a National Literacy and Numeracy Plan that provides a coherent framework for achieving improvement in student literacy and numeracy outcomes. The 1999 Adelaide Declaration of National Goals for Schooling in the Twenty-First Century contains the national literacy and numeracy goal that *students should have attained the skills of numeracy and English literacy, such that every student should be numerate, able to read, write, spell and communicate at an appropriate level.*

In support of the numeracy component of the National Plan, the Australian Government implemented the Numeracy Research and Development Initiative in 2001. This Initiative consisted of two complementary strands – a national project strand and a strategic State and Territories projects strand. The *Developing Computation* Project is one of ten strategic research projects undertaken by State and Territory education authorities across Australia. The purpose of these projects is to investigate a broad range of teaching and learning strategies that lead to improved numeracy outcomes. The *Developing Computation* project was funded by the Australian Government Department of Education, Science and Training and administered by the Tasmanian Department of Education, the Catholic Education Office (Tasmania) and the Association of Independent Schools of Tasmania, in partnership with the University of Tasmania.

The aim of *Developing Computation* was to:

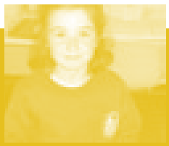
- investigate the effects of strategies that encouraged the use of informal written computation on students' number sense and computational ability
- determine which strategies were most successful
- examine the implications that fostering of informal written calculation methods have for the teaching of formal written algorithms.

The project comprised four stages. Stages 1 and 2 required students and teachers to extend their emphasis on mental computation as preparation for informal written computation. Stage 3 involved exploration and encouragement of informal written algorithms, while in Stage 4, teachers made their own decisions regarding teaching of mental computation, encouragement of informal written methods and teaching of formal written algorithms.

Project impact

It is expected that classroom practice and student numeracy outcomes will be influenced through the compilation and dissemination of:

- documented experiences and recommendations from teachers regarding their approaches to mental, informal written and formal written computation
- evidence of the effects of these approaches on student performance and attitudes, teacher attitudes and parental attitudes
- examples of student work showing:
    - use of paper and pencil in informal and formal ways to aid computation
    - written explanations of their mental strategies
    - stages in the development of informal written methods.


Major outcomes and findings

Student performance in mental computation was compared at the beginning and the end of the project, to provide a measure of the success of the project. There were improvements for all of the three grades and for both genders, indicating that something in the teaching of mental computation had improved over the period. When student attitudes at the start were compared with those at end of the project it was evident that students had increased confidence in mental computation.

Teacher attitudes, beliefs and practices relating to mental and written computation were also surveyed at the start and end of the project and analysis revealed that on almost every item teachers experienced a change of attitude. Teachers expressed their increased confidence in the effectiveness and value of mental computation, work on informal written computation methods, and the delaying of formal written algorithms in the teacher interviews at the end of the project. Interestingly, teachers were not as opposed to holding back from teaching formal written algorithms as researchers had anticipated.

The project team's interviews with and surveys of teachers revealed other indicators of the success of the project. Teachers surveyed on general recommendations and expectations for their classes in the year following the project indicated that they would recommend delaying the teaching of any formal written algorithms in most cases until at least grade 4. All agreed that the concentration on mental computation had greatly increased children's competence and confidence in handling numbers and in understanding place value .

Parents were invited to make comments on their child's progress in numeracy during the project. No negative feedback was received and most comments were from parents who had noticed unexpected progress in their child's numeracy skills.

The results of the project confirm that the introduction of mental computation through a strategies approach is enormously valuable and empowering for children.

This has wider implications for changes in our approach to teaching written computation, which we should not ignore.

1. Background and purpose of the research project

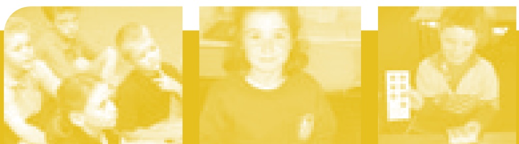
The development of informal methods of written computation in the primary school is encouraged in both the *National Statements on Mathematics for Australian Schools* and in Tasmanian documents such as the *Essential Learnings Outcomes and Standards.*

These documents suggest that young children should be encouraged to explore and invent informal paper-and-pencil methods in order to extend mental computation methods and replace or precede the development of formal written algorithms.

However, there is little research evidence of the effects of developing informal written computation, and no body of practice or systems available to provide guidance for teachers on effective approaches to this development.

The project was designed to support the development of informal written methods in grades 2 to 4 in a small number of schools, to investigate the effects on students' number sense and computational ability, and to analyse critical features of effective teaching strategies.

The project explored the effect on students' numeracy outcomes of using informal written methods when calculating.



2. Objectives and expected outcomes

The four key research questions for this project were:

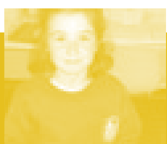
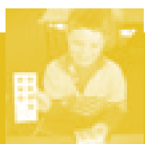
- What is the effect on students' number sense and computational ability of legitimising informal approaches to written calculation through grades 2 to 4?
- What is the effect of using informal written calculation methods on students' numeracy outcomes?
- What classroom strategies and approaches for the encouragement of informal written computation are most effective in developing students' number sense and computational ability?
- What implications does the fostering of informal written calculation methods have for the teaching of formal written algorithms?

The objectives of the project were to:

- investigate the effect of developing informal written computation processes through grades 2 to 4 on student performance, student attitudes to computation, teacher attitudes to computation, and teacher attitudes to classroom assessment of computation
- determine which classroom strategies and approaches for the encouragement of informal written computation are most effective in developing students' number sense and computational ability, and to analyse and explicate critical features of these approaches.

Expected outcomes of the project were:

- reported experiences and recommendations from teachers of grades 2, 3 and 4 in nine primary schools, regarding approaches to mental computation through strategy development, the development of informal written computation, and the place and timing of formal written algorithms
- quantitative and qualitative data regarding the effects of these approaches on student performance and attitudes, teacher attitudes and parental attitudes
- student work samples showing:
- their use of paper and pencil in informal and formal ways to aid computation
- written explanations of their mental strategies
- stages in the development of informal written methods.

3. Research methodology

3.1 Schools and teachers

Nine schools were involved in the two years of the project: five Government schools, two Catholic schools and two Independent schools. The cross-school sector nature of the project supported the selection of a wide range of schools. Criteria involved in selection included schools with different student populations and different socio-cultural contexts; different needs indices and a variety of geographical locations (including isolated schools). This diversity in selection enabled the project to ascertain whether or not informal methods suit particular populations of students and/or particular school contexts.

Further criteria included:

- understanding of, and empathy with, the aims of the project
- willingness of all relevant grade teachers and of the senior staff to be involved in the project
- evidence of prior involvement in modern approaches to mental computation in the school
- presence on the staff of potential Key Teachers (KTs) with relevant skills
- willingness to gain parental support for the project.

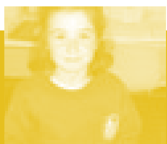
Thirty-seven classroom teachers (all grade 2, 3 and 4 teachers in the nine project schools) were involved in the project.

3.2 Project structure and timeline

The project ran from June 2001 to December 2002 encompassing four distinct stages, as show in Table 1.

Table 1: Four stages of the Developing Computation project

Stage	Duration	Focus
1	June–Dec 2001	Teachers familiarise themselves with the development of mental computation through strategy approach in their classes. No strictures applied regarding teaching of formal written algorithms
2	Jan–May 2002	Development of mental computation through strategy approach with new classes. No teaching of formal written algorithms to these classes.
3	May–Oct 2002	Exploration and encouragement of informal written algorithms. Development of mental computation through strategy approach continues. No teaching of formal written algorithms
4	Oct–Dec 2002	Teachers make their own decisions regarding teaching of mental computation, the encouragement of informal written methods, and the teaching of formal written algorithms


Stages 1 and 2, which occupied well over 50 per cent of the project time, concentrated on mental computation only. While this appeared to leave relatively little time for the central purpose of the project – to look at the development of 'informal written computation' – this long and deliberate introduction was critical, both for children and teachers.

It was critical to concentrate on first developing children's mental computation ability, since the purpose of the project was to observe the process of working on informal written computation with children who have developed strategies for calculating mentally and have a certain level of competence.

For teachers the case was similar. All teachers in the project had previously adopted a traditional approach to the development of written computation, in which the connection with mental computation was almost entirely restricted to the children's use of basic facts (single-digit computation) in computing the partial sums of a multi-digit addition or subtraction (although several of them had already begun to introduce mental computation activities based on strategies and discussion).

This project asked teachers to adopt an approach that was radically different from their traditional approach in at least three fundamental aspects. First, at least as much attention was to be given to children's mental computation of 2-digit numbers as to single-digit computation.

Second, the approach was to emphasise not memorisation of individual facts, but the children's invented or informal strategies for computing the calculations mentally, and their shared oral explanations of their strategies.

Third, when numbers became larger than could be dealt with mentally, instead of learning from the teacher one standard computation method which is both alien to children's natural computation processes and unconnected with their mental methods, children would build on the mental strategies they were using and develop ways of extending these by using pencil and paper.

Stage 1 allowed the project teachers to explore this new approach to mental computation with their existing class in a relaxed way, discussing and appraising the effects with colleagues from their own and other schools. Stage 2, which started at the beginning of the school year, allowed project teachers several months with a fresh class, developing these children's mental computation with an increased confidence and sense of direction. Thus the teachers built their confidence – in their abilities and in the project – in an atmosphere that, while quite new and demanding, was not perceived as threatening.

Teachers' reactions to Stages 1 and 2 were extremely positive. They remarked on the much greater interest and involvement of the children when asked to explain their own methods and an increased competence in discussing and explaining orally. Indeed, when one teacher expressed some doubt about the possible reactions of parents, two teachers from different schools remarked that parents had come up to them asking what they had been doing in maths, as their children had suddenly begun to talk freely about numbers and regularly asked them to give them calculations to do mentally (see Section 4.4). Stages 1 and 2 of the project were therefore perceived by teachers as very rewarding.

Stage 3, however, raised new and potentially threatening aspects for teachers. Stage 3 asked teachers to explore 'informal written computation' (a term which, it would be fair to say, would have been quite unfamiliar to most before the project) and to withhold the teaching of formal written algorithms – a practice which most, if not all, teachers would have considered central to their mathematics program.

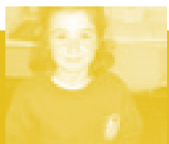
Indeed, this was the reason several schools originally approached to join the project, declined to do so.

Despite some schools declining, however, the holding back from teaching formal written algorithms did not raise anything like the anticipated opposition from those teachers who did participate. Teachers appeared satisfied with the tangible improvements in competence and attitude resulting from the greater concentration on mental computation. They were, in many cases, very aware from previous experience that the traditional approach to teaching standard written algorithms, even with an emphasis on understanding and the use of materials such as multi-base arithmetic blocks (MAB), often caused great difficulties for children. However the encouragement of 'informal written methods', as advocated in the curriculum documents, constituted a quite foreign and unexplored area of classroom practice.

As a result of analysing a wide range of samples of children's informal and formal use of paper and pencil for calculating (*see* Appendix H), teachers developed and used a 6-step process – shown here in Table 2 – for moving from mental computation to informal written computation.

Table 2: Six-step process for moving from mental to informal written computation

Step	**Process**
1	Strengthen children's mental computation with two digit numbers.
2	Encourage children to explain their methods using pencil and paper.
3	Compare, discuss and so refine children's written explanations.
4	Strengthen this method with further calculations of similar difficulty.
5	Extend its use by adapting it to calculate more difficult examples.
6	Consolidate it as an 'understood, secure written method'.


Step 3 is a critical stage. As an example, Figures 1 and 2 show two explanations of computation methods for the same problem from different children.

Figure 1: A wordy but clear and valid explanation of a child's mental method

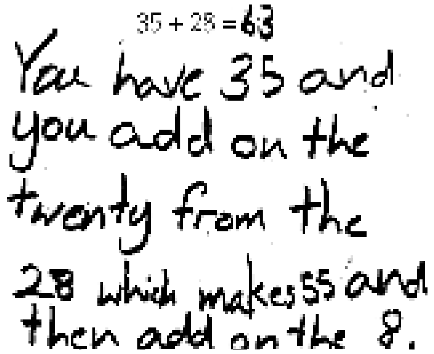
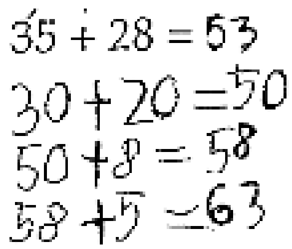


Figure 2: A more concise written explanation of another child's mental method



While both are clear and correct, the form of recording used in Figure 2 is more concise and able to be extended, than that of Figure 1. Children were encouraged to work on the explanations with the help of the teachers, using three criteria for assessing the value of their explanations, based on those proposed by Campbell, Rowan & Suarez (1998): *efficiency* (Is it a concise and effective method?), *validity* (Is it mathematically sound?) and *generalisability* (Could it be used on a wide range of computations, especially with larger numbers?). This provided a scaffold; support for the learners, allowing them to perform complex tasks or processes they might be unable to do alone and proved invaluable in providing teachers with a clear sequence of goals.

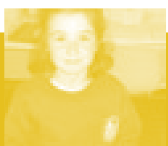
3.3 Assessment data sources

A variety of qualitative and quantitative processes was used. Table 3 summarises the main data sources used, and indicates the timing of their administration within the project.

Details of each of these instruments and processes are given in the Appendices and elsewhere in the report. Analysis and discussion of main findings and conclusions are given below.

Table 3: Details of data sources used for Developing Computation project

Focus	Instrument	Collection points	Comments	**Example**
Teachers	Teacher survey: When are formal written algorithms introduced?	Nov 2001	Teachers indicated when they currently introduced formal written algorithms of addition and subtraction in their classrooms.	Appendix A
Teachers	Teacher survey: Attitudes to mental computation and written computation	Aug 2001 Aug 2002	Teachers were asked to respond to statements using a 5-point Likert scale.	Appendix B
Teachers	Teacher interviews	Jul 2002	Selected teachers were interviewed individually on various aspects of the project.	Appendix C
Teachers	Teacher survey: When to introduce various number topics	Aug 2002	Key Teachers indicated when they would introduce certain number skills in their classrooms as a result of the project.	Appendix D
Teachers	Teacher survey: General recommendations	Oct 2002	Teachers were asked for their general recommendations to other teachers as a result of participation in the project.	Appendix E
Teachers	Teacher survey: Expectations for 2003?	Nov 2002	Teachers were asked to consider what was likely to happen from the project in their schools in 2003, and identify what the system could do to help.	Appendix F
Students	Results of student tests of mental computation	Oct 2001 Sep 2002	All students in the project classrooms were given an identical mental computation test at two points during the project.	Table 4
Students	Student survey: Attitudes to mental computation and written computation	Aug 2001 Aug 2002	Comparison is made of students' response to statements using a 3-point Likert scale, at two stages of the project.	Table 5
Students	Student interviews	Nov 2002	Selected students were interviewed individually.	Appendix G
Parents	Parents' written comments	Nov 2002	Parents from four schools made comments on their children's work during the year.	Section 4.4
Students	Students' work samples	2002	Teachers collected work samples from their students showing three aspects of their work.	Appendix H


4. Outcomes and findings

The project aimed to explore whether, and if so how, it might be possible for teachers to build informal written methods on the foundation of children's developed mental computation strategies. Teachers in grades 2, 3 and 4 of nine Tasmanian schools, representing all three systems, worked on a structured program to strengthen children's mental computation, and then to move as appropriate to build informal written methods on the basis of individual children's mental computation strategies. During the first nine months of the second year of the two-year project, teachers in the project classes undertook not to teach any formal written algorithms to their children.

The project objectives were to investigate, in particular, the impact of informal written computational methods on: student performance, student attitudes, teacher attitudes and parental attitudes, and to determine which strategies for informal written computation have more impact on number sense and computational ability than others. Each of these will now be considered in turn.

Effects on student performance

Every student was tested twice during the project: in October 2001 (towards the end of the first year of the project) and again in September 2002 (towards the end of the second and last year of the project). The mental computation test, which had been developed for students in grades 3 and 4 in a Strategic Partnerships with Industry – Research and Training (SPIRT) project for which the research partner of the current project was chief investigator, consisted of a total of 50 mental computation questions involving the four operations with whole numbers. All questions were administered by the class teachers.

Students listened to the questions (prerecorded on a CD for uniformity) with a 3 second pause after each of the first 20 questions and a 15 second pause after each of the remaining questions. Children were instructed to write the answers only, on an answer sheet. Comparisons were made between the three 2001 and 2002 cohorts (grades 2, 3 and 4) to see whether the 2002 cohorts had gained by a longer and more organised exposure to the project. Results of the tests are given in Table 4.

Performance can be compared by examining the 2002 cohorts of students with the equivalent 2001 cohorts. For example, how did the Year 2 girls in 2002 compare with the Year 2 girls in 2001? Table 4 which contains the relative percentage point gain for correct answers shows that in every case the 2002 cohorts outperformed their 2001 counterparts. Results varied from 2.4 percentage point increase for Year 3 boys to 8.4 percentage point increase for Year 3 girls. The gains appear to be most pronounced in the younger grades. However, as the test was identical for all grades, and thus relatively easier for the older grades, there was likely to be a smaller percentage improvement for older grades as their scores started from a higher base.

While this might not be significant were it a single case, since, for example, the 2001 Year 2 girls are a different cohort from the 2002 Year 2 girls, when in every case through three year groups the 2002 students outperform the 2001 students, it is reasonable to conclude that something in the teaching of mental computation has improved between 2001 and 2002. In 2001, teachers were learning a new approach to mental computation, while their students were having their first introduction to it. In 2002, the grade 3 and 4 students were in their second year of involvement, while their teachers could begin the year with a planned program which they had already trialed.

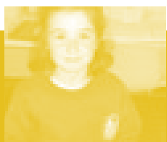
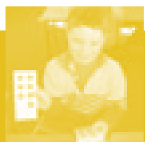




Table 4: Overall percentage correct on the mental computation test by year, grade and gender

Grade	Cohort	2001 % correct	2002 % correct	% point gain
Year 2	Boys	47.3	54.2	6.9
Year 2	Girls	42.5	49.2	6.7
Year 2	Combined	44.9	51.3	6.4
Year 3	Boys	68.5	70.9	2.4
Year 3	Girls	59.7	68.1	8.4
Year 3	Combined	64.3	69.5	5.2
Year 4	Boys	78.8	82.3	3.5
Year 4	Girls	75.7	79.2	3.5
Year 4	Combined	77.1	80.8	3.7

Effects on student attitudes

All children were asked to complete a survey indicating their attitudes and beliefs relating to mental and written computation on two occasions during the project: in August 2001 (near the beginning of the project) and again in August 2002 (towards the end of the project). All teachers of the project classes completed a similar survey form at the same two points (*see* Appendix B).

Table 5 shows the complete list of items included in the two student surveys and a comparison between responses to the first and second survey. The figures show a marked improvement in attitude to mental computation, and increased confidence in their mental computational ability.

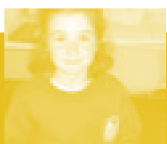
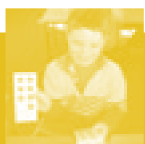
Responses to items 5 and 9 are particularly striking. Item 5 asked students to respond 'Yes', 'No' or 'Don't Know' to the statement: *I often explain to the teacher how I do sums in my head.* The following are percentages of students who said 'Yes' in 2002 compared with those for 2001 (2001 figures in brackets). Results for each age group are: grade 2, 57 (40); grade 3, 57 (34); and grade 4, 39 (27).

Item 9 asked students to respond 'Yes', 'No' or 'Don't Know' to the statement: *My teacher has taught me a proper way to do a sum in my head.* The percentages stating 'Yes' in 2002 (2001 figures in brackets) for each age group are: grade 2, 83 (62); grade 3, 67 (61); and grade 4, 73 (52). The responses of students clearly indicate the change in classroom practice of their teachers (indicated also in the teacher survey (see Appendix B) in the two most crucial elements of the project approach to mental computation. First, students are aware that they are being asked to explain their methods frequently, and second, they are aware that they are being given explicit instruction in mental computation strategies.

The children's increased confidence in their ability to calculate mentally is shown in items19 to 28. In each of

Table 5: Comparison between responses to the first and second student survey (% answering 'Yes')

	Item	Gr 2 2001	Gr 2 2002	Gr 3 2001	Gr 3 2002	Gr 4 2001	Gr 4 2002	ALL 2001	**ALL 2002**
1	I like working out sums in my book.	74	74	67	66	76	65	74	68
2	I like doing sums in my head.	62	71	69	66	62	64	65	67
3	I am good at doing sums in my head.	60	63	57	55	53	55	57	58
4	I am good at working out sums in my book.	69	77	71	69	77	74	74	74
5	I often explain to the teacher how I do sums in my head	40	57	34	57	27	39	33	50
6	I often explain to the teacher how I do sums in my book.	41	56	43	60	43	43	42	52
7	I can add 37 + 26 in my head.	51	49	59	69	69	83	61	69
8	I can add 48 + 27 in my book.	66	75	82	85	90	92	81	85
9	My teacher has taught me a proper way to do a sum in my head.	62	83	61	67	52	73	59	75
10	My teacher has taught me a proper way to do a sum in my book.	69	80	83	81	87	76	80	79
11	You don't need a good memory to be good at doing sums in your head.	43	48	42	34	43	38	43	40
12	It is easier to do sums in my head than to work out answers using paper.	53	53	37	45	28	43	39	47
13	Doing sums in your head is mainly a matter of remembering facts.	56	68	58	64	68	60	61	63
14	I can do sums in my head but I find it hard to explain how I do them.	60	61	69	55	66	65	65	61
15	I can do sums on paper but I don't really understand how I do them.	41	39	28	28	27	17	32	28
16	I like explaining how I do sums in my head.	37	52	30	39	26	27	31	39
17	I like doing mental arithmetic tests	64	70	51	69	57	48	58	61
18	I like doing a lot of sums in my book.	71	70	68	60	66	56	69	61
	I would rather do this sum in my head than in my book.	Head	Head	Head	Head	Head	Head	Head	Head
19	6 + 7	69	80	86	88	93	95	84	89
20	36 + 20	50	57	51	67	72	82	59	70
21	23 + 14	50	53	57	66	70	80	60	67
22	58 + 34	43	47	41	48	45	59	44	52
23	14 – 8	67	68	76	81	79	87	79	85
24	36 – 5	70	77	77	85	87	92	79	85
25	57 – 18	44	46	43	50	44	63	44	54
26	23 x 3	39	41	41	43	44	53	43	46
27	Double 26	47	58	60	68	71	79	61	69
28	Half of 76	35	41	36	41	38	51	37	45

these items, children were given a calculator and asked to respond 'In my head' or 'In my book' to the statement: *I would rather do this sum in my head than in my book.* For each item, taking the three age groups together, there was at least 5 percentage points increase in 2002 in the number of children who responded 'In my head'. The one exception was the only multiplication item; it must be remembered that the project confined itself to addition and subtraction. Particularly striking was the increase in confidence with mental computation of 2-digit numbers by grade 4 children. For example, the percentages of grade 4 children choosing 'In my head' in 2002 (2001 figures in brackets) for selected items are: 23 + 14, 80 (70); 58 + 34, 59 (45); 57 – 18, 63 (44).

Effects on teacher attitudes

The project resulted in a significant change in teacher attitudes towards informal methods of written computation. On two occasions during the project, in August 2001 (near the beginning of the project) and again in August 2002 (towards the end of the project) all project teachers were asked to complete a survey indicating their attitudes, beliefs and practices relating to mental and written computation. The full list of items included in the teacher surveys, together with response data, is given in Appendix B.

On almost every item teachers have experienced a change of attitude, and often a substantial one, as shown in Table 8. The most substantial change in attitude is that teachers indicate that, following their participation in the project, they are more likely to ask their students to explain how they do their mental computations (*see* item 8). This is highly significant since it is a key strategy of the project: it is supported by the student survey, which indicates that students also noticed this change (*see* Table 5). Other important changes between the first and second survey include teachers:

- rating mental computation more important, and written computation slightly less important, and spending more time on the former and considerably less on the latter
- saying that their students enjoy mental computation more and are better at it
- being much less likely to think that there is a 'proper' way to do a written or a mental calculation
- no longer seeing mental computation as mainly a matter of memorisation
- believing that mental computation teaches children more about how numbers work than does written computation
- recognising that calculations are more about connections and relations than about facts and rules.

The apparent changes in their teaching methods (Item 22) appear less dramatic, but that may be because by August 2001 they had already modified or changed their teaching practices to accommodate the project.

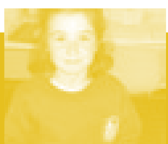
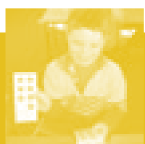
Further evidence of the changes in teacher attitudes, and of their growing confidence in the effectiveness and value of mental computation, work on informal written computation methods, and the delaying of formal written algorithms, are given in Appendix C (Teacher interviews), for example:

Interviewer	What have you found the most enjoyable?
Mr B	Just satisfaction, like I feel like I can finally do something well because I'm seeing results and I'm seeing that enthusiasm from the children – it's just exciting when they're a step ahead of me. I'll be thinking, 'I hope they come up with this strategy today', or, 'I hope they'll see this,' and they do.
Ms J	I have a totally different outlook of how maths is working and why we're teaching it. I really enjoy that I can make maths fun and just to give the children the chance to love it and want to learn is a big step isn't it – and once they've got that, they're like sponges and they'll soak up anything. And I like that there's no right or wrong answers, that we can develop it in our own way and try to get answers in the way that works best for us.

Interviewer	What are your feelings about not teaching the formal algorithm?
Ms B	Fine, because they don't need it. We've got different methods of calculating. If they were really stuck and just couldn't do it, maybe I'd give a formal algorithm and see how they'd go with that. I've felt no need to use it.
Ms G	Great – I think it's the best thing ever because even in grade 6 they didn't understand what they were doing.

Changes in teachers' attitudes are also apparent from the general recommendations of Key Teachers given in Appendix E (Teacher survey: General recommendations). Examples of these are:

Interviewer	Do you recommend teaching mental computation via a strategies approach? Why?
School B	Most definitely! Children are much more confident in solving problems and with their understanding of numbers/place value. I have seen a huge jump in the children's confidence with numbers/problems.
Interviewer	**Has this recommendation changed as a result of your involvement in the project? In what ways?**
School A	Yes. I have faith in the approach because the children have responded so well to being involved in the project.
School C	Yes, definitely. I didn't really teach a range of strategies as such previously. Counting on was probably the main strategy.
School I	Yes – my belief about written algorithms not being introduced until grade 4. How important a good grasp on mental strategies is to understandings of place value.

Interviewer	Do you recommend teaching standard written algorithms? Why?
School B	Not until at least grade 4 or above. Since doing all the strategies and looking at addition and subtraction by splitting, I have found the children are much more confident and excited about maths.
School G	The teaching of standard written algorithms should be taught as just one strategy but only after many, many other strategies are taught – it is just one method amongst many!

Interviewer	Has this recommendation changed as a result of your involvement in the project? In what ways?
School Dii	Very much so. Before this mental computation project I always started the maths program with formal addition and subtraction using tens and units but no carrying or regrouping and I have always progressed from that starting points. This year I haven't done any of this by nearly the end of term 2 and I will do the same next year – no formal algorithms until term 3 or very late term 2. Instead, lots and lots of strategies, games, activities for building understanding of place value and add/sub.
School F	Yes. Previously I taught all the algorithms to brighter grade 2s and grade 3s. I believed that I was teaching for understanding but now I don't think I was. Having taught informal strategies to my class I can see much greater understanding in what the children are doing. Now I don't think they necessarily need algorithms to solve problems, but strategies which work for the individual.

Effects on parental attitudes

Participating teachers and schools were encouraged to inform parents of the project, but the nature and extent of the involvement of parents was left to the individual schools to decide, based on their circumstances and policies. In response to a request from the Research Partner, parents of children in four schools were invited by their teachers to make any comments they wished on their child's progress in numeracy during the year. In all, twenty-seven written comments were received. No negative feedback was received.

Most comments were from parents who had noticed unexpected progress in their child.

- [My child] is very *keen to try her numeracy. She thoroughly enjoys it and has certainly advanced dramatically.*
- *[My child]'s understanding of mathematical concepts and operations has improved notably this year. [She] has gained confidence in her ability to complete mathematical problem.*
- *Her maths development has been quite dramatic with simple maths done much faster in her head.*
- *My child... has shown fantastic development in numeracy this year. She often computes aloud to us, showing her mental strategies and seems to be able to work out incredibly hard sums (problems) without needing algorithms. We are thrilled with her progress. Being a primary teacher of many years' experience, I can really appreciate the level [my child] is working at. She has a much better mental ability than many of the Gr 5/6s I teach.*
- *[My child] has been actively applying mental maths in everyday situations. This has demonstrated her heightened interest in maths as a result of this work.*
- *[My child] appears to have developed skills and even enjoyment in mental computation that we have not witnessed in our older child.*
- *As a parent, I didn't understand it at first. But my daughter has come leaps and bounds. She loves maths. She does maths work at home and then she rubs it out and does it again.*

- *I have seen lots of [my child]'s maths work and it is brilliant. He enjoys doing it as well and that makes a difference.*

Six parents commented that they had noticed progress, but were unable to say if it was unusual, or was due to the project.

- *One year in a child's life is a long time. Yes, I have seen some good progress in [my child's] comprehension and progress but I can't say whether it is more than one would hope for over this period.*
- *We have not been conscious of any 'unusual' progress with [our child's] mental computation. However we are well aware that he is very capable in this area.*

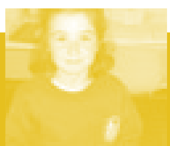
Several parents who had attended parent information sessions also made interesting and perceptive comments.

- *As a mother to three boys, two already in high school and both, always having done reasonably well in maths, I must admit that when I was invited to the information evening of mental computation I did think to myself, maths is maths, what else is there to learn. However, once I heard the presentation I found it to be very interesting, informative and logical. It just simply seemed to make sense. I think perhaps my two older boys tended to add in this way naturally and were moulded into formal structured maths in class.*
- *The idea that it is becoming acceptable for children to not have to work out their sums in a formalised way is wonderful.*
- *It was interesting that in a small room full of people, we all had such varying ways of calculating simple sums. I do believe that this type of thinking is so much easier when dealing with calculations in everyday life, i.e. shopping, banking. To be able to think 'on your feet' is so much more natural than writing down every transaction or every price of every product on a grocery list.*

While the number of responses is only small, and may represent a biased sample, it is nevertheless unusual for parents to give such positive written responses, in a curriculum area where uninformed researchers might predict a quite different reaction.

That the response was unusual, was borne out by the responses of some teachers in the teacher interviews (*see* Appendix C), for example:

Interviewer	Have you had any reaction from parents?
Ms E	Oh, they're fine. They all accepted that. We explained that as part of this we wouldn't be doing a lot of written maths, but that's OK… I've just started my second round of parent–teachers so I really haven't had enough chance to see any more yet, but generally they seem very happy with what's happened.
Ms G	We've been well supported in this school by [our Key teacher] and I knew a little bit last year what was going on because the staff was buzzing with it and so it was on my agenda for my parent–teacher evening. I had 100 per cent of parents turn up and I very clearly explained that this is what we're doing. Their reaction was fantastic.


Strategies with impact

A major focus of the project was to find answers to the question: Which strategies for informal written computation have more impact on number sense and computational ability, than others? As previously explained, the actual time during the project devoted to fostering informal mental methods was inevitably brief. So it was not considered possible or desirable to leave teachers, for whom even the idea of developing other than formal written algorithms had formerly been unthinkable, to start from scratch in devising their own pathways.

The project therefore suggested a 6-step pathway from mental computation to informal written methods, with the understanding that teachers could adapt or depart from these steps as they felt necessary or desirable. In the event, no teachers reported any major departure from the structure, which was not surprising in a situation in which all were treading new ground. However the comments of teachers described above, and given in detail in the Appendices, indicate that a move from mental to written computation along the lines suggested is both feasible and of major benefit.

It must be left to further, and longer-term research projects to chart and evaluate in more detail alternative routes along the computation highway. This project has shown that the route is traversable and worthwhile.

Summary of findings

Throughout the project researchers regularly monitored and discussed teachers' views, reactions and recommendations, with particular attention to those of the Key Teacher(s) in each school.

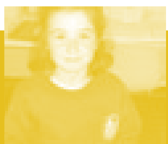
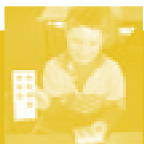
The project produced a number of significant findings.

- All teachers in the project agreed that the concentration on mental computation had greatly increased children's competence and confidence in handling numbers and in understanding place value. Most schools saw Prep or Kindergarten as the best time to start this approach for most children.
- All teachers in the project would now recommend delaying the teaching of any formal written algorithms – in most cases until at least grade 4. All agreed that their views had been considerably modified by the project. Whereas before most said that they would have advocated the introduction of standard written algorithms from grades 1 or 2, they now suggested grade 4 as more appropriate. They saw the increased understanding of numbers and place value acquired through concentrating on mental computation provided a much more secure basis for the later introduction of the written algorithms.
- All teachers agreed on the benefits of developing informal written methods as a bridge between mental and formal written methods. They stated that it helped children to develop 'understood ways' of dealing with larger numbers, that seeing their thinking strategies in writing helped them to clarify their mental strategies, and that the presence of written work helped teachers to 'see' children's thinking and to work on misunderstandings.
- Teachers can readily assimilate the new strategy-based approach into their teaching provided that a structured approach is provided which gives clear sequences and goals.
- Teachers can use a 6-step process as a basis for developing informal written methods with children.

5. Impact on classroom practice and numeracy

The project has the capacity to affect classroom practice in significant and lasting ways.

- It confirms the value of the move away from the teaching of unique rule-based written algorithms for each of the four operations with whole numbers as the basis of number work in the primary school
- It confirms the value of an early number program based on conceptual understanding and a strategy-based approach to mental computation
- It provides one practical framework for approaching informal written methods (as advocated in curriculum documents) by building on children's mental computation skills and understandings
- It provides validity to professional discussions by teachers, curriculum providers and the community for discussing the place and value of formal written algorithms in the primary classroom
- It confirms the efficacy of ways of working in research-based curriculum development with teachers that value and draw on the distinctive professional knowledge and experience of classroom teachers.

6. Other issues and implications

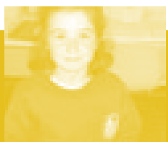
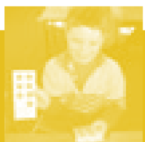
There was a need to develop in teachers as well as children, an approach to mental computation through strategies with which most were unfamiliar. As there was only a six-month period in which informal written methods were explored and developed some teachers were moving at a faster rate than they might have wished in implementing the changes. Though the preferred development was to introduce informal before any formal methods, this was not possible as, inevitably, some of the project children had already been introduced to formal written methods before the start of the project.

The project was not designed to find or propose a specific solution, but rather to explore with teachers the consequences for themselves and their children of moving in this direction, and to provide some insights from their experiences that may help other schools as they face the same situation.

The project outcomes send out two clear messages. First, they confirm the recommendations quoted earlier, that the introduction of mental computation through a strategies approach is enormously valuable and empowering for children. Second, they make clear that this change has wider implications for changes in our approach to teaching written computation, which we should not ignore.

Finally, there were two significant benefits that arose from this project and its funding arrangements, and which were remarked on frequently by teachers and systems during the project. All found it of enormous value to have had:

- the opportunity for collaboration in a research project between education systems and a university research partner
- the opportunity for collaboration between schools in the State, Catholic and Independent systems.

7. Research papers arising from the project

McIntosh, AJ 2003, *Approaching computation through mental computation.* Paper delivered at the Nineteenth Biennial Conference of the Australian Association of Mathematics Teachers, University of Queensland, 13–17 January 2003.

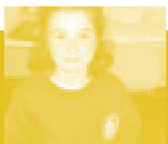
McIntosh, AJ 2003, *Developing number sense through mental and informal written computation.* Paper delivered at ICASE 2003, the World Conference on Science and Technology Education, Penang, 7–10 April 2003.

McIntosh, AJ 2003, *Reorienting the teaching of computation.* Invited paper delivered at the Midsummer World Mathematics Conference, Lingatan, Sweden, 19–25 June 2003.

McIntosh, AJ 2003, *A coherent approach to mental and written computation.* Keynote address delivered at the Making Sense of Numeracy Conference, Queensland University of Technology, 2 August 2003.

Moane, K & McKenzie, G 2002, *Mental munchies – putting theory into practice.* Proceedings of the Springboards into Numeracy: Proceedings of the National Numeracy Conference, Hobart 4–5 October 2002. Adelaide: Australian Association of Mathematics Teachers Inc.

McIntosh, AJ 2002, *Developing informal written computation.* Paper delivered at the annual conference of the Australian Association for Research in Education, Brisbane, 1–5 December 2002.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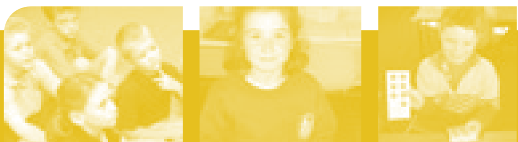


9. Appendices

Appendix A

Teacher survey: When are formal written algorithms introduced?

In November 2001, all project teachers were asked the grade at which they currently introduced written algorithms in addition and subtraction in order to gain baseline data regarding current practice with regard to written computation. Teachers displayed an understandable reluctance to be precise, in order to allow for the diversity of their students. General results for the majority of children are shown in Table 7.

Table 7: Grade of introduction of written algorithms for addition and subtraction

Algorithm	Digits	Grade
Addition	2-digit	2
	3-digit	2 or 3
	4-digit	3 or 4
Subtraction	2-digit	2 or 3
	3-digit	3 or 4
	4-digit	4

The table shows that formal algorithms for the operations of addition and subtraction for 2-, 3- and 4-digit numbers are normally introduced in the project schools in grades 2 to 4, thus confirming that the grades chosen for the project do coincide with the years when most students normally meet these formal written algorithms.

However it was clear, both from many of the work samples provided by teachers, and comments of the teachers themselves, that many of the grade 2 children in the project had already met some formal written algorithms, whether at home or at school.

Appendix B

Teacher survey: Attitudes to mental computation and written computation

On two occasions during the project, in August 2001 (near the beginning of the project) and again in August 2002 (towards the end of the project), all thirty-seven project teachers were asked to complete a survey indicating their attitudes, beliefs and practices relating to mental and written computation.

Table 8 shows a comparison between mean responses to the first and second survey. In questions 1 and 2, teachers were asked to rate their agreement with the statement on a scale of 1 (least agreement) to 10 (most agreement). Questions 3a and 3b asked teachers to indicate the average time (minutes per week) they devoted to mental and written computation. In items 4 to 19, teachers were asked to rate their agreement with the statement on a scale of 1 (strongly disagree) to 5 (strongly agree). Items 22a to 22h asked teachers to indicate the percentage of the time they devoted to mental computation for each of a number of activities.

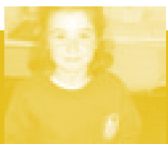
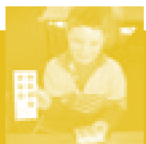




Table 8: Comparison between mean responses to the first and second teacher survey.

	Item	1st survey	2nd survey	% change
1	How important do you rate the teaching of mental computation (1 to 10)?	8.4	9.3	0.9
2	How important do you rate the teaching of written computation (1 to 10)?	8.1	7.8	-0.3
3a	On average, number of minutes per week you teach mental computation?	77.2	92.2	15.0
3b	On average, number of minutes per week you teach written computation?	154.3	86.6	-67.7
		5 = strongly agree, 1 = strongly disagree		
4	Most of my students enjoy mental computation.	2.2	4.2	2.0
5	Most of my students enjoy written computation.	2.3	3.8	1.5
6	Most of my students are good at mental computation.	2.7	3.8	1.1
7	Most of my students are good at written computation.	2.6	3.5	0.9
8	I often ask students to explain how they do their mental computations.	2.0	4.5	2.5
9	I often ask students to explain how they do their written computations.	2.1	3.9	1.8
10	I often ask my students to add two 2-digit numbers mentally.	2.8	4.3	1.5
11	I often ask my students to add two 2-digit numbers using written methods.	2.2	3.6	1.4
12	There is a best way to do any mental computation.	4.2	2.0	-2.2
13	There is a best way to do any written computation.	3.8	2.2	-1.6
14	You don't need a good memory to be good at mental computation.	3.0	3.3	0.3
15	WC teaches more about how numbers work than MC.	3.6	1.8	-1.8
16	Mental computation is mainly a matter of memorising facts.	3.9	1.9	-2.0
17	For some children memorising number facts and rules is more efficient than understanding.	3.1	2.3	-0.8
18	Calculations are more about facts and rules than about connections and relations.	3.9	1.7	-2.2
19	Calculations are about connections and relations.	2.0	4.5	2.5
	Assign a figure to indicate the percentage of time you give to each of the following, when teaching mental computation (total 100):			
22a	Speed and accuracy tests	13.1	7.8	-5.3
22b	Games for practising facts	18.2	21.4	3.2
22c	Reciting tables	4.7	5.8	1.1
22d	Silent individual practice	7.7	4.9	-2.8
22e	Children discussing strategies	15.3	17.2	1.9
22f	Teaching a particular strategy	20.5	19.1	-1.4
22g	Activities to develop strategies	20.6	23.0	2.4
22h	Other	0.8	0.8	0.0

Appendix C

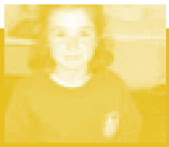
Teacher interviews

In July 2002, thirteen teachers from four of the project schools were interviewed; the outcomes of those interviews are shown in Table 9.

Table 9: Project teachers interviewed in July 2002

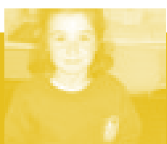
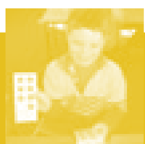
School	Teacher	Class taught in 2002	Class taught in 2001
School 1	Ms A	Grade 4	Grade 4
	Ms B	Grade 3	Grade 3
	Ms C	Grade 2	K and Grade 3
School 2	Mr D	Grade 2	Grade 3/4
	Ms E	Grade 2	Prep
	Ms F	Grade 3	Grade 2/3
	Ms G	Grade 3	Grade 6
	Ms H	Grade 4	Grade 4
School 3	Ms I	Grade 3/4	Grade 3/4
	Ms J	Grade 1/2	Grade 1/2
School 4	Ms K	Grade 3	Grade 1/2
	Mr L	Grade 3/4	Grade 3/4
	Ms M	Grade 3/4	Grade 3/4

The interviews were semi-structured; the list of questions that were to be asked was given to each of the teachers a day or two before the interviews. After teachers answered each of the main questions, further questions were asked to clarify or extend the teachers' responses. All interviews were videotaped and transcribed for analysis. The following are extracts from some responses.

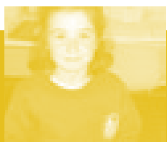
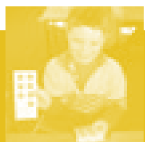

	What do you think the children have enjoyed most?
Ms A	The games and those types of activities, and the sums… the feeling of success – they have really shown their ability to do these computations.
Mr B	The games, definitely. They filled in a survey not long ago and all of them wrote that they loved the games and just about every lesson they say, 'Can we start with a game?' They love the games which reinforce the number facts.
Ms C	I think for some of them it's giving them a strategy that is more secure – a way of working it out that makes sense. They can actually work it out in their head without having to memorise that this has to go with this… it makes more sense to them.
Ms G	I think the games. They know that they're doing maths but it's a fun way of doing maths. I think first their own satisfaction because they know that they're improving and they know that they're understanding and it's not a foreign language.
Ms I	I think they're enjoying the activities because when they're throwing dice they think it's fun, that it's a game, and then at the end when we discuss it we say we did our addition, our subtraction, we were using our co-operative skills, our social skills, developing our oral language, listening skills, and they can see that it's not just a game, there's a whole point to it.
Ms K	The feeling of success, even for my better mathematicians.
Mr L	Well, I think that they like to verbalise how they've solved it and get accolades for that, because once they come up with something they often say, 'Oh, I never thought of it that way. That's an interesting way.' And then we discuss my way and their way, and they get a lot from that.
Ms M	Well, they love that whole group talk and sharing their strategies, and when I stop that and they need to move on, they say, 'No, no just one more. But I've got a different way!' They're so keen to share the strategies, and they love using the Make 100, Make 50, Make 20 boards and I've found lots of other games that we can adapt and do that sort of thing, and they're really enjoying it. I'm actually getting them a lot of the time saying, 'When are we going to do real maths?' I say, 'This is real maths!'

	What have you found the most enjoyable?
Ms A	All of it. I've loved it – absolutely loved it. It makes the number strand not such a grind and you can actually pick out what their weaknesses are.
Mr B	Just satisfaction, like I feel like I can finally do something well because I'm seeing results and I'm seeing that enthusiasm from the children – it's just exciting when they're a step ahead of me. I'll be thinking, 'I hope they come up with this strategy today,' or, 'I hope they'll see this,' and they do.
Ms C	Just making it more fun for the children, and seeing that they will come to the realisation that they have a really good knowledge of this so that later on anything you put in front of them they'll be willing to work out rather than just saying, 'I haven't seen this before, I can't do it'.
Ms E	It's making me re-think how to teach – not just, 'This is the way to do it'. Because they come up with some wonderful ideas, things I could never think of, and obviously if they're thinking of it, it's a child thing where they can teach other children. Not, 'This is the way we do it. This is the way I've got to get through'. But taking time and saying how would you like to do it.
Ms G	I've enjoyed the children's enthusiasm, their joy at actually doing maths, their preparedness to have a go. I think I've enjoyed using a different strategy – and I must admit it's different from anything I've done in the past and I've been teaching for a long time. And it's fairly challenging but also been enjoyable.
Ms I	A new way of thinking.... I'm trying to get them to understand – I think we're nearly there – that we're away from thinking, 'I've got to do it the teacher's way" to, 'The teacher's way is one way, and this is one way, and that's one way'.... And they're also getting confident enough now to know that 'My way is right'.
Ms J	I have a totally different outlook of how maths is working and why we're teaching it. I really enjoy that I can make maths fun and just to give the children the chance to love it and want to learn is a big step isn't it – and once they've got that, they're like sponges and they'll soak up anything. And I like that there's no right or wrong answers, that we can develop it in our own way and try to get answers in the way that works best for us.
Ms K	I like having the sequential structure, starting at the beginning and going right through, knowing that you've covered everything ... I'm finding links and we're going back and using strategies we've learned before, and things like that. Yes – just the structure of the program has been really good.

	What has it done for children's confidence?
Ms A	Oh – [My class this year is] quite a lot different – a lot more confident – they don't wait, they go straight to it – don't wait to find out how to do it, what to do, they just do it. [And] when they're working, you'll see them looking at something and do a double take – they'll think 'Oh, that couldn't possibly be right' – especially the weaker children…
Ms E	Because I say to them, 'It doesn't matter if it's right or wrong, tell me how you did it,' they're more willing to have a go – the slower ones, I think are more willing to have a go at it. The pressure's off, it doesn't matter if it's wrong. Whereas if you gave them the normal way – I'm talking about 2-digit addition here – they'd get confused with the carrying the tens and things.
Ms F	They're all very keen to participate in anything to do with maths, and not just in the numeracy area. I find that there's a keenness, 'Oh good, it's maths!' when they come back from recess.
Ms G	Oh, absolutely, yes. They've been more positive. And as soon as we start maths they say 'Great' and they're already finding partners. Another thing I've found which is absolutely wonderful is that the really good children will team up with an average child and you've got a lot of teaching going on and they can explain things I can't, in their own way, and I guess you learn more when you're teaching. I think you learn 70 per cent of what you teach and I think the peer tutoring – and that's just happened, I hadn't really planned this, it's just happened.
Ms J	Well it's really helping and I've noticed it's moving into other strands of our curriculum. For example we were skipping with our Daily PE the other day and the children were counting by 2s and counting by 5s how many skips they were doing and they were adding those up really quickly – I can see that they're starting to use it now in other subjects so they're developing their confidence – it's good you can see it working.
Ms M	Oh – it's increased their confidence – just the fact that they're willing to sit there and take risks in front of everyone, and that's across the board. All abilities will have a go.

	What are your feelings about not teaching the formal algorithm?
Ms B	Fine, because they don't need it. We've got different methods of calculating. If they were really stuck and just couldn't do it, maybe I'd give a formal algorithm and see how they'd go with that. I've felt no need to use it.
Mr D	I think as we come more to the end of Term 2 and go into Term 3 it's likely to be essential that we now build on that [mental] computational learning that's happened and I think that will work magnificently in the children's development. I'm very happy with the way it's gone.
Ms G	Great – I think it's the best thing ever because even in grade 6 they didn't understand what they were doing
Ms H	That's been a bit of a worry but we've skirted around it by doing a lot of other things – a lot of space work, measurement, had them involved in measurement.
Ms I	As I said to you before, grade 3 you start 24 add 26 first day back. Now that we've been using these strategies I don't want to start because my children are all splitting the tens, they're working from the tens, and I'm really concerned how am I going to start saying, '24 add 27 – right, 4 add 7 is 11, put down the 1 and carry the 1'. It's nothing to do with what they've been doing so successfully and so well.
Ms J	It's easier for me because I have a grade 1/2. It's not going to worry me so much because I'm not teaching them. In the beginning I was a bit worried because we've always been told that's part of what we teach and that's how we do it. I'm really seeing now that the understanding is so much more important than just drilling off an answer with no idea whatsoever. So I think it's better to delay and to hold off. I think I would still teach the other method just as another way and the children can then decide what strategy does work best for them and whether to adopt that one or not.

	From October onwards, you'll make your own decisions about how you'll teach them the written algorithm. What, at this stage, do you think you're likely to do?
Ms A	If I have to I will – if their next year's teacher says yes – but I will prefer not to do it.
Ms B	I'll probably consult the grade 4 teacher, before I make a decision. She might want to continue just letting the children use their own kinds of strategies that hopefully will be refined by then and I think that'd be great. I personally think they could start algorithms in grade 5, but that's just personal.
Ms E	Probably the middle to top part of the class won't need much teaching, they'll have worked out their own strategies, and I think just the bottom part I'll have to say, 'This is how you have to add up'.
Ms G	I think there will come a time when we have to do it, obviously, but I don't think I want to do it just yet … I'd leave it till the end of the year I think, 'til the children are really understanding and have reached the level that I consider satisfactory…. I don't think it's a task for grade 3. I think that comes with maturity as well.
Ms I	I think I'll have to because the two grade 5/6 teachers will be horrified if they can't come up with setting it out vertically. I would like to have a talk to them and ask, 'Would you mind if I didn't?' Perhaps they can leave grade 3 /4 having really good mental strategies.

	Have you had any reaction from parents?
Ms E	Oh, they're fine. They all accepted that. We explained that as part of this we wouldn't be doing a lot of written maths, but that's OK… I've just started my second round of parent–teachers so I really haven't had enough chance to see any more yet, but generally they seem very happy with what's happened.
Ms G	We've been well supported in this school by [our Key teacher] and I knew a little bit last year what was going on because the staff was buzzing with it and so it was on my agenda for my parent–teacher evening. I had 100 per cent of parents turn up and I very clearly explained that this is what we're doing. Their reaction was fantastic.

Appendix D

Teacher survey: When to introduce various number topics

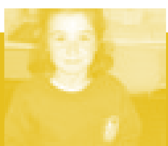
Towards the end of the project, in August 2002, Key Teachers were asked what grade (if any at all) they would now recommend for the introduction of elements of mental and written computation to children of upper, middle and lower ability. A summary of their recommendations is given in Table 10.

Table 10: Summary of recommendations from Key Teachers for the grade at which to introduce elements of mental and written computation to children of upper, middle and lower ability

Topic	Upper ability	Middle ability	Lower ability
Counting	K	K	K
Addition using objects and counting	K	K, possibly P	K, possibly P
Subtraction using objects and counting	K, possibly P	K, possibly P	K, possibly P
Addn/subn mental strategies for basic facts other than counting	P	P, possibly 1	P, possibly 1
Memorisation of addn/subn basic facts	P or for some 1	I, or for some P	1, possibly P or 2
Formal written addn algorithm (1 digit)	Not at all, but if so, P	Not at all, but if so, P, possibly 1	Not at all, but if so possibly P or 1
Formal written subn algorithm (1 digit)	Not at all, but if so, P	Not at all, but if so, P, possibly 1	Not at all, but if so possibly P or 1
2-digit addn strategies (mental)	1	1, or for some 2	2, or for some 1, possibly 3
2-digit subn strategies (mental)	1, or for some 2	2, or for some 1	2, or for some 1, possibly 3
2/3 digit informal written addn methods	2, possibly 1	2	2, or for some 3, possibly 4
2/3 digit informal written subn methods	2, possibly 1	2	2, or for some 3, possibly 4
Formal written addn algorithm (2 digit)	4 or 3, possibly not at all or 5	Probably 4, possibly 3 or 5 or not at all	4, possibly 3 or 5 or not at all
Formal written subn algorithm (2 digit)	4 or 3, possibly not at all or 5	4, possibly 3 or 5 or not at all	4, possibly 3 or 5 or not at all
Formal written addn algorithm (3 digit)	4, possibly 3 or 5	4, possibly 3 or 5	4, possibly 5 or 3
Formal written subn algorithm (3 digit)	4, possibly 3 or 5	4, possibly 3 or 5	4, possibly 5 or 3

Key to summary of the 10 responses

All 10 teachers:	Indicated by text in bold, on its own in column
5+ teachers:	Indicated by text in bold
4 or 3 teachers:	Text is 'probably', or 'for some'
2 or 1 teachers:	Text is 'possibly'


Appendix E

Teacher survey: General recommendations

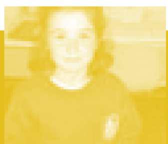
In October 2002, almost at the end of the project, Key Teachers were asked to give general recommendations for the introduction of certain topics, based on their experience in the project.

The following are the questions and responses of the ten Key Teachers from the nine schools (one school had two Key Teachers).

1a. Do you recommend teaching mental computation via a strategies approach? Why?	
School A	Yes. Children need to master single-digit strategies then move towards 2-digit numbers. Also, move from addition to subtraction.
School B	Most definitely! Children are much more confident in solving problems and with their understanding of numbers/place value. I have seen a huge jump in the children's confidence with numbers/problems.
School C	Yes. If children are exposed to a number of strategies they will discover the method that suits them (that they will understand best). We can also get a good idea of their understanding by observing their informal written methods, which should reveal their strategies.
School Di	Yes. It broadens their thinking and makes them aware there is no right or wrong answer, just the most efficient way for you.
School Dii	Definitely! Children must have a clear understanding of the numbers they are using and what they are really doing with those numbers. By introducing a variety of strategies children should have a clear understanding of the processes of addition and subtraction. Children need lots of experimenting and working with visual aids.
School E	Yes. Children must be taught a range of strategies in order for them to be efficient and effective users of numbers. A strategic approach allows a sequential teaching and a learning program appropriate to children's stage of development. A strategies approach allows for lots of discussion/sharing and working with concrete materials.
School F	It makes sense, so definitely. You can progress through the different strategies (friends of 10, doubles, etc) and the children like working on each new idea. They seem to like to understand how to do the problems and then enjoy activities which consolidate the strategies. As a teacher I enjoy having a series of strategies to follow so that you ensure the children are learning all that is necessary.

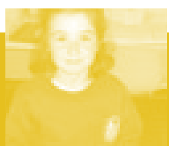
1a. Do you recommend teaching mental computation via a strategies approach? Why?	
School G	Yes. The teacher can build on one strategy after another, almost safely assuming that once all the strategies are taught, students will have a bank of methods to rely on to solve problems mentally. Students do not learn to mentally compute by osmosis, they need to work through many methods and develop efficient ways to calculate problems.
School H	Absolutely – the very strongly organised and sequential package we have been following has had a fantastic flow on effect – confidence within the students, so that they are able to apply the strategies to more complex algorithms and to successfully get a solution.
School I	Yes, definitely. I think we need to empower children to become independent learners and teaching strategies gives a sound base for them to draw understandings of various strategies when working mentally. Gives a strong base for understanding place value and communicating understandings.

1b. From what grade?	
School A	Depends on the child. When they a good understanding of basic number facts. Prep? Year 1.
School B	Kindergarten beginning/main focus in Prep.
School C	We can start with simple strategies from Prep, e.g. doubling to 5.
School Di	Grade 1. However, it really depends on the individual child, i.e. they need to have mastered all the sub-skills first, i.e. counting, add and subtract, using counters, etc.
School Dii	K onwards. Children need visual aids and experimenting from the very beginning of their numeracy experiences.
School E	Kindergarten. I believe children are ready to explore numbers, manipulate concrete materials, count, order, etc. at the age of 4.
School F	Late kinder/prep.
School G	From kindergarten because students then become used to discussing strategies and their thinking.
School H	Prep – to give students from the very beginning some very solid grounding in order to arm them with great strategies to build on.
School I	Needs to start right at kindergarten.


1c. Has this recommendation changed as a result of your involvement in the project? In what ways?	
School A	Yes. I have faith in the approach because the children have responded so well to being involved in the project.
School B	Yes, it has changed due to my involvement in the project. I have focused on teaching the strategies and reinforcing these, instead of just doing algorithms straight away. I wouldn't have thought to do facts of 100 etc. before.
School C	Yes, definitely. I didn't really teach a range of strategies as such previously. Counting on was probably the main strategy.
School Di	No.
School Dii	I have always used aids etc but much more so since I have been involved in the MCP. The project has given me a clearer understanding of the ways in which children learn; it has been an excellent review. The project has provided us with activities that can be readily used because they are user friendly and we have been given clear explanations of their value and use. The response of children has added value.
School E	I have always been an advocate of mental computation however my involvement in the project has taught me a range of strategies, the purpose of mental computation, practical ideas, developmental sequence and on the whole encouraged my teaching practice to be specific, meaningful and explicit.
School F	Yes. I think I was starting to change my thinkings as I was becoming involved in Count Me in Too (CMIT), but the introduction of the strategies/teaching understandings has been strengthened and I think it is important to teach for understanding as soon as you introduce addition and subtraction. When you introduce addition it makes sense to use the strategies as a focus. I would introduce addition and subtraction in prep.
School G	I am now more convinced that the explicit teaching of strategies for mental computation is essential. I am now an advocate for the systematic teaching, revising and discussing of strategies and thinking.
School H	Yes. Introducing the project at year 2 in the project has been one of the few negatives. Grade 2s quickly become very competent at using the suggested strategies and so were able to apply them and prep would be a very good stage to begin to get this happening much more quickly.
School I	Yes – my belief about written algorithms not being introduced until grade 4. How important a good grasp on mental strategies are to understandings of place value.

	2a. Do you recommend teaching standard written algorithms? Why?
School A	Yes. Because the children need to know how to apply their number skills with these problems. Needs to be delayed.
School B	Not until at least grade 4 or above. Since doing all the strategies and looking at addition and subtraction by splitting and I have found the children are much more confident and excited about maths
School C	Not before grade 5. Students are able to use their own written strategies which make sense to them.
School Di	Yes. It gives the child another option and ways of thinking. It is essential, however, they have mastered all the sub-skills first, e.g. place value. I would not enforce that they use it. I would ask them to use the most efficient method for them.
School Dii	Yes. As children use the various strategies for add/sub they are aiming towards the most efficient method and in the end formed written algorithms are the most efficient – but only if the understanding has developed so that there is no confusion.
School E	Yes! For some children this may be the best way!! I think when children are asked to work with large numbers an algorithm can provide them with structure.
School F	I think by about grade 4, children should be introduced to the standard algorithms so that if they choose to use this method they can. But I believe that they need a strong understanding of mental strategies because they will need to be able to do things in their heads as well. Furthermore using informal methods often makes more sense to the children so I would encourage them to use them if they choose best.
School G	The teaching of standard written algorithms should be taught as just one strategy but only after many, many other strategies are taught – it is just one method amongst many!
School H	Yes. I think there comes a time when big numbers need a formal written algorithm – addition or subtraction of money, of large measures, etc.
School I	Yes, by grade 4 most children will need some form of efficient recording in a formal arrangement. However, this should only be introduced if the child has strengthened mental strategies and a good understanding of mental strategies and a good understanding of place value. Also children should be able to choose what method formal or informal of recording that suits them.

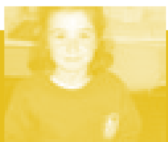
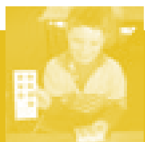




2b. From what grade? Why?	
School A	Year 4 as above. When dealing with numbers in 1000s place value is clearer.
School B	About grade 4. Just because from what I have heard and seen this seems a good age.
School C	From grade 5. It may be difficult to add/sub numbers beyond 3 digit mentally, and without a S.W.A.
School Di	Grade 3 – to ensure they have mastered sub-skills first.
School Dii	Grade 5/6 definitely. I think by term 3 of grade 4 most children should be ready for the formal written algorithm. I know this from the children in my present class.
School E	Grade 4
School F	Grade 4
School G	Perhaps late year 4 for 2-digit and 3-digit addition and subtraction without regrouping – leave that to year 5.
School H	From Grade 4. Students in year 3 in the project have demonstrated in their work samples that they are able to uniformly calculate lots of problems in very mathematically valid ways and are excited, very involved and love maths.
School I	Grade 4

2c. Has this recommendation changed as a result of your involvement in the project? In what ways?	
School A	Yes. Foundation years have moved too quickly into written algorithms before children have mastered place value.
School B	Yes. I would have been teaching the standard written algorithm in grade 2 before but now I would focus on all the strategies before.
School C	Yes, definitely. Previously I was using the standard written algorithm (SWA). for 2-digit add/sub in grade 2. I realise now that the deep understanding just wasn't there, as there were many errors and ridiculous answers.
School Di	Yes. Beforehand I would have introduced written algorithms in grade 1 and grade 2, believing they would pick up the idea of place value. What's happening without teaching the specific sub-skills first.
School Dii	Very much so. Before this MCP I always started the maths program with formal and addition and subtraction using tens and units but no carrying or regrouping and I have always progressed from that starting points. This year I haven't done any of this by nearly the end of term 2 and I will do the same next year – no formal algorithms until term 3 or very late term 2. Instead of lots and lots of strategies, games, activities for building understanding of place value and add/sub.

2c. Has this recommendation changed as a result of your involvement in the project? In what ways?	
School E	Yes, I would have taught the standard way at a much earlier stage (perhaps grade 1 or 2) and in doing so not adequately provided a rich, varied and developmentally appropriate maths program.
School F	Yes. Previously I taught all the algorithms to brighter grade 2s and grade 3s. I believed that I was teaching for understanding but now I don't think I was. Having taught informal strategies to my class I can see much greater understanding in what the children are doing. Now I don't think they necessarily need algorithms to solve problems, but strategies which work for the individual.
School G	Yes, previously I would have recommended vertical addition (algorithms) to be taught from year 2 – firstly one digit, then 2 digit but now I do not see the benefit of setting out addition and subtraction in formal way for young children – there are many other methods to use which develop understanding!
School H	Yes. Have previously accepted that it is okay to teach formal methods of calculating and have thought that teaching the process, even though the understanding may remain a mystery. Am now sure that understanding of the process, and the added advantage of the place value falling naturally into place as being much more important.

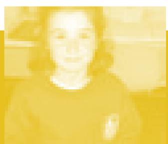
3a. Do you recommend children develop some informal written methods? Why?	
School A	Yes. Because it accepts different strategies and will therefore be more relevant to the child. Children's understanding of mathematical concepts is strengthened.
School B	Most definitely! I would never have thought of teaching all the different strategies first but they definitely are very supportive of addition with double digits and children can do sums so quickly in their heads.
School C	Yes. These methods developed by the children themselves make sense to them. They can apply these methods to higher numbers.
School Di	Yes, To ensure they understand the process – to eliminate mistakes, i.e. typing to store and remember facts in head.
School Dii	Yes. I encouraged children to jot down anything that was in their minds so that I could understand when they were thinking. I was amazed at their work. From reading their explanations I could easily 'see' their thinking and progress from there. I was also amazed at the variety of thinking from 22 children in one class for one problem.

3a. Do you recommend children develop some informal written methods? Why?	
School E	Yes. Not all children can hold information mentally. Asking children to record their workings allows teachers to assess progress, intervene, plan appropriately and share workings with colleagues. Informal written methods are personal/individualised/meaningful to the student.
School F	Yes. Informal methods (written) allow the children to be able to write down what they are doing so that when they come across larger numbers they are able to work with them rather than having to work with numbers in their head. It also helps for the children to use in helping explain what they are doing.
School G	Yes, the written informal methods help the child to develop suitable methods, it allows the child to keep track of their thinking and to not lose sight of what they are doing.
School H	Yes, experimenting, listening to and sharing strategies used by other students and then finding the best one to suit at the time, after trying other suggestions is an excellent way to learn and eventually evolve to the most efficient method.
School I	Very strongly recommend the children developing written informal methods of computation as this enables them to see their thinking strategies in writing and helps clarify the mental strategies they use in solving a problem.

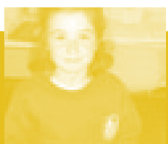
3b. From what grade? Why?	
School A	Prep onwards.
School B	Prep. I would like to see the doubles/ten facts and the counting on stages.
School C	As soon as possible, e.g. preps can represent through pictures/symbols.
School Di	1 or 2. After they have mastered place value.
School Dii	K/P – not sure from my lack of knowledge and experience of their work. But definitely from grade 1 onwards. Children need to visualise their thinking and that of other children. It helps children to ‘share” their thinking.
School E	Prep – kinder program is very much exploratory. Prep children are developmentally ready to record.
School F	Grade 2 – as soon as you introduce mental 2-digit addition/subtraction.
School G	The students should be writing (recording) their methods from an early age – perhaps prep.
School H	Grade 1 – feel sure that grade 1 can begin to use such things as a blank number line as a means of informally recording their method of working.
School I	I think students should record thinking strategies from prep and grade 1 as these informal strategies again help clarify and reinforce their thinking strategies when computating mentally.

3c. Has this recommendation changed as a result of your involvement in the project? In what ways?	
School A	Yes. I value the importance of mastering the foundation skills.
School B	Yes. Because I have been shown all the different ways to teach these strategies and other teachers have shared with me ways to consolidate this knowledge. As well, I have found parents supportive which is excellent.
School C	Yes. Previously I thought the correct way was the horizontal or vertical formal written method.
School Di	Yes. Before the project, I didn't really do much informal written methods, and tended to encourage quick mental calculations or formal written methods.
School Dii	For the last 5 years, I had had a 'how did you do that?' session in mental computation. But working on the project has reinforced just how important this strategy is for the development of children's understanding – and for my understanding of their thinking.
School E	No.
School F	Yes. I haven't taught informal written methods – that is I haven't encouraged children to use their own methods and explain their understandings/workings.
School G	Previously I believed that all students should be taught to record methods in the same way. This project has completely changed my outlook on this – that the thinking and development of strategies to suit each child are more important than 'setting out' in a prescribed way.
School H	Yes, I don't think I have ever thought about uniformly representing different ways of solving calculations before the project – have previously focused on teaching only a formal approach.
School I	Yes, my changed thinking for a number of years has been reinforced. I believe lots of mental strategies which are explicitly taught are of enormous benefit and working out how problems are solved in an informal way helps children clarify and reinforce understanding of what they are thinking mentally.


4a. Do you recommend delaying the teaching of standard written algorithms? Why/why not? If so, by how much compared with present practice?	
School A	Yes. Children should master the basic concepts before learning the 'monkey trick' without understanding what it means.
School B	Yes, I recommend delaying the teaching. I have found that with written algorithm children really just learn a process but don't necessarily need to have an understanding of the numbers.
School C	Yes. Delay till grade 5 as numbers go beyond 3-digit and the S.W.A. may be more efficient. Informal written methods will show if children have developed a good understanding.
School Di	Yes. To ensure sub skills are mastered first and to make sure they have the time to explore and experiment using a range of strategies.
School Dii	Yes. The understanding of the processes must be solid and this can be achieved by using a variety of strategies and by adopting the informal written methods. If these understandings are solid, then the understandings of standard written algorithms should be more easily developed.
School E	Yes, by 2 years to grade 3. Why delay? To ensure children have time to practise using a range of strategies, have solid understanding and confidence manipulating numbers and are reliable with place value.
School F	Previously I taught algorithms to children in grade 2, now I believe that it should not be introduced until grade 4 when the children have a great understanding of place value/ working with all numbers. Only when they have understanding/strategies of their own should children be encouraged to think and determine what is best for them
School G	Delaying of standard written algorithms is a good idea. It allows for a greater period of time to be devoted to the development of a variety of mental strategies. Delay this teaching for up to 2 years to late Year 4.
School H	Yes. Would recommend that formal written algorithm be delayed until grade 4. Grade 3's samples strongly suggest that even brightest students are happy, not really needing a formal method, when they can just as quickly solve in an informal way and the mathematical understanding demonstrated is fantastic.
School I	Yes, especially if a child has not developed strong mental strategies and understandings of place value, it would only cause complications for the child if they were introduced to standard written recording before lots of ground work has been understood.

4b. Has this recommendation changed as a result of your involvement in the project? Why, and in what ways?	
School A	Yes, because children have a better understanding of numbers and place value as a result of mastering single then double mental strategies.
School B	Most definitely. I am finding that children are more aware of what a reasonable answer would be and they have a better awareness of the values instead of calling all numbers as single digits as they do in carrying.
School C	Yes. The project has helped me recognise that formal written algorithms do not show whether the children understand. Informal written methods reveal the level of understanding.
School Dii	Yes. I would delay standard written algorithms until late term 2/term 3, grade 4. The MCP has been very effective in my having made this decision for my children. I would never have delayed standard written algorithms without having seen and heard the evidenced from the MCP.
School E	Yes. Greater emphasis must be placed on mental computation as adults use this form for the majority of number.
School F	Yes, introduction to other ways of teaching addition and subtraction has allowed me to see the power of other methods and the need not to teach algorithms at such an early age.
School G	Yes. I have not begun to think about teaching any formal written methods (standard algorithms) with my year 2 and I hope that they may have at least another year to explore the many addition/subtraction strategies which work for each of them.
School H	Yes. Formal approach has previously been my way – teaching it from the bbd, drumming it in, step by step, over and over, lots of examples, so that process is taught but no emphasis on real understanding.
School I	Yes, previously I only had a limited perception of the ways to teach mental thinking and its importance. Through being involved in this project and being introduced to the research and thinking of the importance of mental strategies I have a better understanding of where I am going and what I believe. My teaching is much more interesting and relevant and enjoyable. This has been very enriching and exciting for me and my teaching.


Appendix F

Teacher survey: Expectations for 2003

Towards the end of the project, in November 2002, Key Teachers were asked what they though likely to happen in their schools in 2003, the year following the project, and what kind of system action or support would be useful. The following is a summary of the responses of each school.

School 1	We plan on informing all teachers in the school about the project at the beginning of the year. All resources from the project are being copied for teachers not involved so far. A parent information night is planned. Outside professional support would be very helpful.
School 2	Plan to extend the project throughout the school. Want to see a much higher profile for mental computation from prep and grade 1. Those in the project are capable of leading school-wide change in teaching style and content. Sending printed materials does not change practice. PD is needed, involving practicing teachers as providers. Help with planning parent sessions is needed.
School 3	We intend to continue with developing children's mental strategies as we have been doing. We hope to involve kindergarten and prep. Support in bringing other teachers in would be valuable, and spreading of the materials to all teachers. Help with an information session for parents would be useful.
School 4	We will continue in grades 2 – 4, including informal written methods, and prep onwards will be using Count Me In Too. It is likely that grade 5 will introduce formal written algorithms. More PD for staff is essential. We will have another information session for parents.
School 5	I have negotiated with the principal to lead a whole school PD day during the first week of the year to provide all staff with a package for mental computation with accompanying games and resources. We have made a decision to strengthen mental computation throughout the school. Grades 2 and 3 will continue with the project, and delay the teaching of the formal written method. We hope that prep and grade 1s will collaborate. We have financial support from our parents' and friends' funds and have identified resources to be acquired. We hope that the Catholic system will disseminate the project materials throughout schools. PD will be needed to support new teachers.
School 6	PD led by project teachers for K–4 staff. Consistency of approach in K–4 in developing mental computation strategies. Parent communication and sharing. Collation and coordination of resources. Needs: University pre-service courses need greater emphasis on teaching mental computation. Assistance with ways of involving upper primary and lower secondary teachers. PD for parents. Consideration of the best way to develop and present the package for teachers.

School 7	The participating teachers will continue with the program as in 2002. We will endeavour to share our skills and resources with colleagues. However not being a whole school project makes it difficult time-wise to convince and involve other teachers. Group emailing to share our ideas and resources with other project schools would be helpful, and continued access to information regarding new ideas/research/resources.
School 8	Only one staff member who has been in the project will be at the school next year. Before I leave, I want to run a session for new staff and upper primary teachers. I should also like to run a session for teachers in the district. Perhaps the project Key Teachers could be used to share the package and its philosophy with a number of new schools.
School 9	We are extremely pleased with the project and will be extending it into prep and up to grade 4. We should like to see continued PD and sharing sessions available for all staff and schools. The project package will be very valuable.

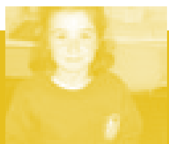
Appendix G

Student interviews

Towards the end of the project, in November 2002, twelve children – six grade 4 children from each of two schools – were interviewed individually for about 20 minutes each. Grade 4 children were chosen as they were most likely to have been exposed to the project the longest. The teachers were asked to choose children who, in their opinion, were somewhat above average in ability, about average, and somewhat below average, but avoiding extremes of ability. All interviews were videotaped and transcribed.

The questions included a number of calculations for which students were given time to calculate and then asked for their solution strategies. Other questions probed their feelings about mental and written computation, about their preference for being 'shown a method' as opposed to being allowed to calculate in any way they liked, and their opinions about the value of learning to calculate mentally. The full list of questions is given below.

1. Do you like doing mental arithmetic/sums in your head? Why/Why not?
2. Do you prefer doing sums in your head or on paper? Why?
3. How did you do these items?
   (Items asked were: 8 + 6, 14 – 7, 20 + 70, 36 – 8, 140 – 60, 58 + 34, 57 – 18).
4. Have you done more mental computation this year? Why do you think that? Is that good? Why?
5. Does your teacher ask you to explain how you do MC? Is that good?


6. Do you think MC is useful to you in school? When? Why?
7. Do you think MC is useful to you out of school? When? Why?
8. What is the hardest sum you could do in your head?
9. How can you work out sums that are too difficult to do just in your head?
10. Do you prefer to be told exactly how to do things in maths or to be asked how you could do them?

The questions in which students were asked to voice their opinions yielded few answers of interest. The children appeared to accept whatever happened in their classrooms as the reality, and explained what happened but did not comment. The exception arose in their responses to question 10. Though one child's responses were inaudible, ten of the other eleven children, said that they preferred the challenge of being able to use any method they liked.

One child said: 'Because I think it helps me to learn by myself and helps me to think instead of showing me'. However, two of these ten children said that, although they preferred to work things out for themselves, they thought they learned more when the teacher told them how to do something.

The eleventh child started by replying, 'It doesn't really worry me. Just however', but, when pressed said, '[I prefer] Miss D telling us... just so if we have to do it another time we can do it the same way'.

Only three of the children said that they preferred to do things on paper rather than in their heads. Reasons for preferring paper were:

- because it's easier, you don't have to remember the sum.
- because you can write it down and think better.
- because you can see your working out and you can see the numbers that you've got.

Reasons for preferring to calculate in the head included:

- Sometimes when I do it on paper I get confused.
- I think it's just a bit easier working in my head.
- Because I just like doing it that way.
- Because I find it's easier, I like it and it's just comfortable for me to do it that way.

Uses for 'sums' outside school, for those children who could think of any, were almost entirely restricted to shopping, with one mention of banks and one of scores in sport.

The children did show that they commanded a good range of strategies, although it is not possible to say to what extent this is attributable to their involvement in the project. They were also very articulate in their explanations even when their thinking was incorrect or opaque. Examples of the variety of responses include:

8 + 6 *Well you just plus two that would equal ten and there was a four left so you just add four.*

I did the six and six is twelve and then add the two.

I just think in my head, umm... six is made up of three twos and eight and two is ten and the other two twos equal four and ten and four is fourteen.

Eight and eight is sixteen, take...take two is fourteen.

14 – 7 *I know that seven plus seven is fourteen so I can take seven away from fourteen.*

Fifteen take eight is seven.

My doubles, seven add seven is fourteen.

Well, four and three makes seven so I minussed the four and then I minussed the three.

36 – 8 *Well I put a... you know how you put it down and you put the thirty-six take eight and that's what I done, and then I went, um, six.........and eight, you can't take six from eight [yeah] so you take that and [writing in the air as she explains] you make that into two and then you put the one onto that and it's sixteen and take eight and got it on and done that way.*

57 – 18 *Umm, I had the fifty-seven and I took the ten away from the eighteen and took it away from the fifty seven and that got forty-seven and I took eight and I got thirty-nine.*

I took the eight and then the ten. When I took the eight off I had forty-nine and then I got thirty-nine.

58 + 34 *Fifty add thirty is eighty, and then add eighty-eight and add four is ninety-two.*

Umm, I took the eight and the four away and fifty and thirty equal eighty and then I put an eight on the eighty I knew that the eight and two was ten and that made it ninety and I had two left over and that made it ninety-two.

I added on my thirty on to the fifty, which gave me eighty, and I added on my eight and my four which gave me twelve and then I added my twelve on to my eighty which gave me ninety-two.

It is worth quoting in full one child's response to a calculation: it shows the complexity of thinking that children can achieve – and of course the resulting difficulties for the teacher. Whether or not the child's thinking is in fact logical, it does show that exchanges of this kind are by no means low-level or superficial: one wonders where else in mathematics children's thinking is engaged and stretched in this way.

I Yes, very good then. What about thirty-six take eight?

S *Twenty eight*

I Very good, how did you do that?

S *Umm, well, I knew that eight and two equals ten so I knew that if I just take four away from the six and took two away from the thirty-eight.*

I Hang on. You obviously know what you are doing but I couldn't quite get it.

S *Umm... I took away four of the six and that left me with thirty and I still have two left to take away and I knew that um, ten take two is eight.*

I You said you take four of the six, and that leaves you thirty?

S *No that leaves me two.*

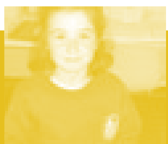
I Oh, OK.

S *Thirty-six take four of the six equals thirty-two.*

I Right.

S *Yeah, and then I take two away [yeah] and that gives me... and then I take eight... sorry...out of thirty-six, ... um ... I split the... I took six of the eight out and that left me thirty and I took the leftovers, that was two, away and I go twenty-eight.*

I Ah, I see. Sometimes it's quite difficult to follow what someone's thinking.


Appendix H

Student work samples

On three occasions, project teachers were asked to gather examples of the ways in which their children used paper, whether formally or informally, to help them in performing calculations. On the first occasion the intention was mainly a fact gathering exercise to inform the researcher: do children use paper other than for formal algorithms, and if so how? The second collection was intended to rouse teachers' interest in the ways children used paper and pencil, to attempt a classification of the various types of use, and generally to assist teachers to accept that it might be possible for children to develop paper-and-pencil methods other than the formal written algorithm. These two collections had therefore a mainly professional development aim.

The third data collection was to provide a snapshot of three stages in children's development of informal written methods in collaboration with their teachers, corresponding to stages 2, 3 and 5 of the sequence (shown in Table 2 of the report). These three 'snapshots' were collected over a period of no more than four weeks for the purpose of discussion amongst the project teachers – a much shorter timeframe than that envisaged as normal practice. Figures 3, 4 and 5 show an example of the work of one child at each of these three stages.

Each of the three collections of data formed the basis of a short booklet, which was distributed to all project teachers and discussed as part of the professional development sessions.

Figure 3: A child's written explanation of his mental calculation method

Figure 4: The same child recording a similar calculation after 'conferencing'

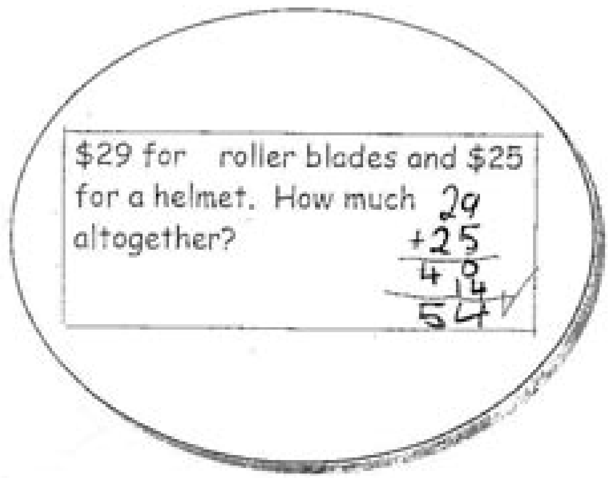


Figure 5: The same child using a similar method to compute a more difficult calculation